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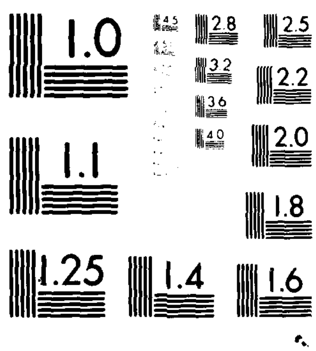

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STRUCTURES REPORT 382

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by

D. G. FORD

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SUMMARY

The continuity equation has been applied to fatigue crack models in which all cracks begin together to form a family of smooth trajectories proceeding at different rates. When the rates at a particular crack length are log-Normally distributed, it is possible to estimate the effect of the mean and variance of crack life in a reliability situation. For a logarithmic variance 0·1 of crack rates, the standard deviation of life is reduced by 14% approximately.

This crack model has been applied to a previous one-crack analysis to allow randomness in both initiation and cracking.

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16. *ABSTRACT*

The continuity equation has been applied to a fatigue crack model in which all cracks begin together to form a family of smooth trajectories proceeding at different rates. When the rates at a particular crack length are log-Normally distributed, it is possible to estimate the effect on the mean and variance of crack life in a reliability situation. For a logarithmic variance 0·1 of crack rates, the standard deviation of life is reduced by 14% approximately.

This model has been applied to a previous one-crack analysis to allow randomness in both initiation and cracking.

CONTENTS

A

NOTATION

$A \mid t$	Random crack length conditional upon time t
a, $(a \mid t)$	Crack length (at time t)
$a_0 = a \mid t_0$	Length of initial cracks
$f = f(a \mid t)$	Crack length density
f	Probability density function according to subscript
g, h	Heuristic constants, Section 4.3
G, $G(t_a)$	Defined by Equation (4.5)
H, $H(t_a)$	$= E(t_a) - ut_a$
M_a	Moment generating function of median cracking life
$M_A(u)$	Moment generating function of crack life
$M_t(u)$	MGF of initiation time
$M_\phi(u)$	MGF of total life
$r(a) = r(t_a)$	Risk or hazard rate
$r_0 = r(0)$	Hijack risk
R, $R(a, t)$	Generic cracking rate
$R_0(a) = R_0(t_a)$	Median growth rate
$R_0 = R_0(0)$	Initial median growth rate
t	Overall time or life
t_a	Median crack growth time
t_A	t_a with random cracking
$t_0 = t - t_a$	Initiation
T_a	Median life to runaway cracking
u	Transform variable for MGFs
X	Random factor for particular cracks
x	Particular value of X
$\phi(t)$	Density of life
$\phi_A(t_A)$	Density of crack time with random rates
μ_{ra}	rth moment of median crack life
ξ	$= \ln x$
σ^2	Variance of x
$\sigma_a{}^2$	Variance of median crack life

1. INTRODUCTION

Until now, the general theory of structural fatigue[1,2] has treated the growth of cracks in structures with two distinct stages—viz. crack initiation or damage, followed by crack growth. The idealisation implied in those papers is that variability is introduced by random initiation time, while structural interactions are related to the growth of cracks. In reality, the second assumption is fairly accurate but there is randomness in the crack growth exhibited by nominally identical structures. So far, allowing for this has defied analysis for multiply-cracked structures; the present report extends the previous work to allow random cracking rates in single-crack models.

It begins with an exact treatment of the crack growth models of Payne[3] or Hooke[4] which is then combined with random initial lives. Previous results for life and moment generating function are generalised and there is some discussion of the low order effects of random crack rates.

2. DISTRIBUTION OF CRACKING LIFE

This replaces the reliability type of life density[1] previously found for one-crack models with deterministic cracking, and indeed it will transpire that the present result is obtainable from that model by simple substitution. Before beginning, however, the randomness in crack rate must be described more closely.

2.1 Nature of Random Cracking

For the earlier deterministic cracks,[1] the possible trajectories were a group of curves *obtainable from one another* by translation. These were indexed by the initiation time t_0, but all satisfied the rate $da/dt = R(a)$, a being crack length and t time, while R is a single-valued function.

In practice R is random, as in the first trajectory of Figure 1. Each growth increment during cycling is random and there is also an overall materials effect acting for every load applied to a given structure. The first of these two types of randomness corresponds to some stochastic differential or difference equation and leads to the second order diffusion term in the Fokker–Planck equation. The second overall type of variability allows smooth crack growth for any particular structure but variations between them as shown on the right of the figure.

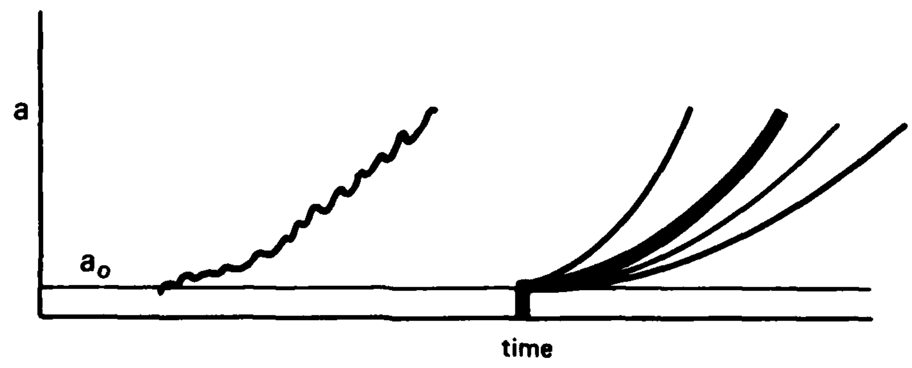

FIG. 1 NATURE OF CRACK GROWTH

If diffusion is ignored but overall variations are allowed for, the fatigue model—though different—is still deterministic and follows the previous continuity equation

$$\frac{Df}{Dt} = \frac{\partial f}{\partial t} + R(a, t)\frac{\partial f}{\partial a}$$

$$= -f(\operatorname{div} R(a, t) + r(a)) \tag{2.1}$$

for crack length density $f(a|t)$ given time t.

2.2 Rate Divergence

We now assume that for a crack beginning at t_0, $da/dt = XR_0(a)$ which replaced $R(a, t)$ above. Here X determines the overall crack rate and is a function of a, t and t_0. Without knowledge of $a|t$, X is random with density $f_X(x)$, log-Normal where necessary; the particular trajectory corresponding to any x is a transformation of X to $A|t$.

Then the divergence

$$\frac{\partial R}{\partial a} = x\frac{dR_0}{da} + R_0(a)\frac{\partial x}{\partial a}$$

with t and t_0 fixed.

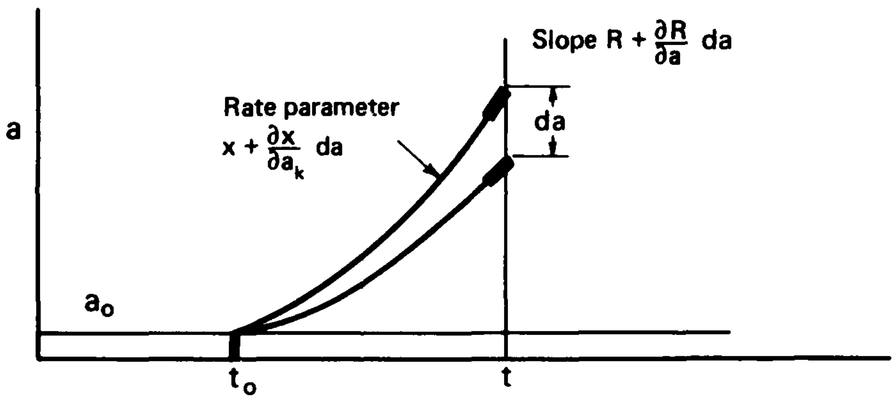

FIG. 2 RATE DIVERGENCE

For any crack da/dt integrates to

$$x = \frac{1}{t - t_0}\int_{a_0}^{a} \frac{da}{R_0(a)}$$

whence

$$\frac{\partial x}{\partial a} = 1/(t - t_0)R_0(a) \tag{2.2}$$

and

$$\frac{\partial R}{\partial a} = x\frac{dR_0}{da} + \frac{1}{t - t_0}. \tag{2.3}$$

This is illustrated by Figure 2.

2.3 Crack Length Density

With the present model, the characteristic equations of (2.1) are

$$\frac{-df}{f(xR_0'(a)+r(a)+1/(t-t_0))} = dt = \frac{da}{xR_0(a)}. \tag{2.4}$$

Along the crack trajectories (for which x is constant)

$$\begin{aligned} -d\log f &= \frac{R_0'(a)}{R_0(a)}da + \frac{r(a)}{xR_0(a)}da + \frac{da}{x(t-t_0)R_0(a)} \\ &= d\log R_0(a) + \frac{r(a)}{xR_0(a)}da + \frac{da}{t_a R_0(a)}. \end{aligned}$$

Integrating,

$$\log f = A - \log R_0(a) - \frac{1}{x}\int_{a_0}^{a} \frac{r(a)}{R_0(a)}da - \int_{a_0}^{a} \frac{da}{t_a R_0(a)}.$$

Now let t_a be the growth time of the median crack ($x=1$) and change the crack length variable a to t_a, the time for the median crack to reach the same size.

Then

$$\log f = A - \log R_0(a) - \frac{1}{x}\int_0^{t_a} r(c)dc - \log t_a. \tag{2.5}$$

This is singular at initiation $t_a = 0$ because all cracks here begin at the one point (t_0, a_0) and (2.5) indicates an infinite density corresponding to the concentrated probability that $a|t_0 = a_0$. However, the equation suggests that we express f in terms of a modified "density" ft_a.

Thus,

$$f(a|t)t_a = \frac{A}{R_0(t_a)}\exp\left(-\frac{1}{x}\int_0^{t_a} r(c)dc\right). \tag{2.6}$$

where A is constant along each trajectory, as is x. Consider another median growth time $x\tau$ for which

$$fx\tau = \frac{A}{R_0(x\tau)}\exp\left(-\frac{1}{x}\int_0^{x\tau} r(c)dc\right).$$

Substituting for A in (2.6) leads to

$$f(a|t)t_a = R_0(x\tau)\frac{f(a|\tau, x)x\tau}{R_0(t_a)}\exp\left(-\frac{1}{x}\int_{x\tau}^{t_a} r(c)dc\right). \tag{2.7}$$

Here, of course, $t_a = x(t-t_0)$ and since x is constant along a characteristic, it may be cancelled from each side to give

$$f.(t-t_0) = \tau f(a|\tau, x)\frac{R_0}{R_0(x(t-t_0))}\exp\left(-\frac{1}{x}\int_0^{xt} r(c)dc\right). \tag{2.8}$$

where $R_0 = R_0(0)$, the initial growth rate.

2.3.1 Boundary Conditions

These will be expressed in terms of $f(a|t)t_a$. Being supported only at (t_0, a_0), they must be regarded asymptotically. So far we have not used the assumed density $f_X(x)$, nor any assumptions about asymptotic behaviour.

As in Figure 3 we assume a finite initial crack rate (unless $x = 0$) already implied by our use of $R_0 = R_0(0)$. Together with continuity, this presumes asymptotically linear initial growth consistent with the two-stage fatigue assumption of a fixed finite initial crack length. In welded structures, where immediate initiation is more plausible, the same behaviour agrees with the assumption of a pre-existing flaw, restricted, however, to a constant size.

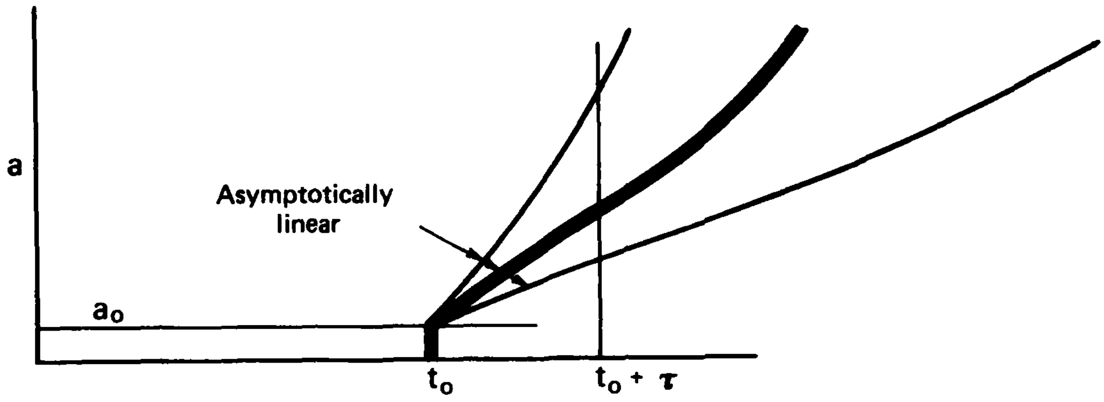

FIG. 3 APPROACH TO BOUNDARY CONDITIONS

As $\tau \to 0$

$$a \sim xR_0\tau$$

and the asymptotic crack length density

$$f(a\,\tau) = f_X(x)/R_0\tau,$$

so that

$$\tau f(a\,\tau) \sim f_X(x)/R_0$$

and (2.8) becomes

$$f(a\,t,t_0) = \frac{f_X(x)}{(t-t_0)R_0(x(t-t_0))}\exp\left(-\frac{1}{x}\int_0^{x_t} r(c)dc\right)$$

$$\frac{xf_X(x)}{t_aR_0(t_a)}\exp\left(-\frac{1}{x}\int_0^{t_a} r(c)dc\right) \tag{2.9}$$

2.4 Density of Crack Life

At a given time, the total rate of attrition is

$$\phi(t\,t_0) \int_{a_0}^{\infty} \frac{f_X(x)}{(t-t_0)R_0(t_a)} r(t_a)\exp\left(-\frac{1}{x}\int_0^{t_a} r(c)dc\right) da \tag{2.10}$$

remembering that median growth for time $t_a = x(t-t_0)$ produces the crack (a,t). Again, t_a is the natural variable of integration and we obtain

$$\phi(t|t_0) = \int_0^{\infty} \frac{f_X(t_a/(t-t_0))}{t-t_0} r(t_a) \exp\left(-\frac{t-t_0}{t_a}\int_0^{t_a} r dc\right) dt_a \tag{2.11}$$

When cracks are small one expects this to be consistent with attrition at the constant risk r_0; we show that this *is* r_0.

Near $t = t_0$, the reliability type density from (2.11)

$$r(t_a) \exp(\ldots) \to r_0 \exp(-r_0(t-t_0)) \tag{2.12}$$

for all cracks from (t_0, a_0), i.e. all values of x. This factor, the desired asymptotic behaviour, may be taken outside the integration. The remaining integral is asymptotically one since

$$f_X(t_a/(t-t_0)) = f_X(x),$$

a density function.

3. RUNAWAY CRACKS

These will be treated by telescoping as in Reference 1. The present assumption is that the median crack "runs away" when $t_a = T_a$ so that other cracks are runaways after the actual times T_a/X. Mathematically this means that $r(T_a+) \to \infty$ so that the risk integrated past T_a is also infinite.

3.1 Reduction of Life Range

Equation (2.11) may then be written

$$\left(\int_0^{T_a-} + \int_{T_a-}^{T_a+} + \int_{T_a+}^{\infty}\right) \frac{f_X(x)}{t-t_0} r(t_a) \exp\left(-\frac{1}{x}\int_0^{t_a} r dc\right) dt_a \tag{3.1}$$

and we first consider the third integral in the limit as risk uniformly increases.

Thus, let $R > r(t_a) > \rho$ when $t_a > T_a$ so that

$$R.(t-t_0) > \frac{t-t_0}{t_a}\int_0^{t_a} r(c)dc > \rho(t-t_0) \qquad (x = t_a/(t-t_0))$$

in which case

$$e^{-R.(t-t_0)} < \text{exponential term} < e^{-\rho(t-t_0)}$$

Let $f_X(x)/(t-t_0)$ be bounded. We require an upper bound for

$$\int_{T_a+}^{\infty} f_X r e^{-\int r\, dc/x} \frac{dt_a}{t-t_0} < \int_{T_a+}^{\infty} f_X\left(\frac{t_a}{t-t_0}\right) R e^{-\rho(t-t_0)} \frac{dt_a}{t-t_0}$$

$$< R e^{-\rho(T_a-t_0)}\{1 - F_X(T_a/(t-t_0))\}.$$

If now there are finite constants C, d such that $R < C\rho^d$ then

$$R e^{-\rho(t_a-t_0)} < C_\rho{}^d e^{-\rho(t_a-t_0)}$$

$$\to 0 \qquad \text{as } \rho \to \infty$$

Thus if $C\rho^d > r(t_a|t_a > T_a) > \rho$ the third integral in (3.1) vanishes as $r(T_a+) \to \infty$.

3.2 Telescoping

Now consider the second term

$$\frac{1}{t-t_0}\int_{T_a-}^{T_a+} f_X\left(\frac{t_a}{t-t_0}\right) r(t_a) \exp\left(-\frac{t-t_0}{t_a}\int_0^{t_a} r(c)dc\right) dt_a$$

$$= \frac{f_X(T_a/(t-t_0))}{t-t_0} \int_{T_a^-}^{T_a} r(t_a) \exp\left(-\frac{1}{x}\int_0^{t_a} r dc\right) dt_a + O(\Delta T), \tag{3.2}$$

using the mean value theorem.

Apart from the factor $1/x$ the integrand above is a probability density of life appropriate to a risk $r(c)$. To develop this idea consider a particular crack trajectory, along which x is constant. Then reliability theory indicates the conditional life distribution

$$\Phi_T(t \mid t_0, x) = 1 - \exp\left(-\int_{t_0}^{t} r(x(t'-t_0))dt'\right)$$

$$= 1 - \exp\left(-\frac{1}{x}\int_0^{t_a} r(c)dc\right) \tag{3.3}$$

with $c = x(t'-t_0)$ and $t_a = x(t-t_0)$. This has the density derivative

$$\phi_T(t \mid t_0, x) = x r(t_a) \exp\left(-\frac{1}{x}\int_0^{t_a} r(c)dc\right), \tag{3.4}$$

the integrand of (3.2) apart from x, here equal to $(t-t_0)/T_a + O(\Delta T)$. Thus (3.3) provides the integral in (3.2) and with runaway cracks (3.3) is one if $t_a > T_a$. With all substitutions the limit of (3.1) is

$$\phi(t \mid t_0) = \int_0^{T_a} \frac{f_X(x)}{t-t_0} r(t_a) \exp\left(-\frac{1}{x}\int_0^{t_a} r(c)dc\right) dt_a$$

$$= \frac{1}{T_a} f_X\left(\frac{T_a}{t-t_0}\right) \exp\left(-\frac{t-t_0}{T_a}\int_0^{T_a} r(c)dc\right). \tag{3.5}$$

In this generalisation of (2.11) T_a, the median time to runaway cracking, is effectively a constant parameter. The second term, the density of runaways, is similar to but not the same as the corresponding term with deterministic cracking; the same idea of concentrated crack life attrition still applies.

4. LIFE DISTRIBUTION

From Reference 1, the life distribution for translated cracks

$$\phi(t) = r_0 e^{-r_0 t}(1 - F_0(t)) + \int_0^{T_a} e^{-r_0 t_0} f_0(t_0) dF_a(t-t_0)$$

$$+ e^{-r_0(t-T_a)} f_0(t-T_a) \exp\left(-\int_0^{T_a} r(c)dc\right). \tag{4.1}$$

where if $t_a < t - t_0$

$$dF_a/dt_a = r(t_a) \exp\left(-\int_0^{t_a} r(c)dc\right) \tag{4.1A}$$

from reliability theory applied to crack life.

The moment generating function here is

$$M_\phi(u) = M_i(u - r_0)M_a(u) + \frac{r_0}{u-r_0}(M_i(u - r_0) - 1) \tag{4.2}$$

where $M_i(u)$ and $M_a(u)$ generate moments of initial life and crack time.

If M_a is replaced by another function reflecting the outcome of attrition on randomly cracking structures from $t = 0$, the unique inversion of (4.2) indicates that $\phi(t)$ is similarly generalised by generalising the reliability density (4.1A), the inverse of the new $M_a(u)$.

For the different but deterministic crack model above, this replacement density is (3.5), which already includes any runaway cracks so that the third term in (4.1) is subsumed into the second.

For completeness, the generalised life distribution will be found directly. For this, one needs $f(a'\, t, t_0)$ which is (2.9) in the present instance. The formula (3.5) is also the density of $t_A = t - t_0$, the crack part of the total life. Equation (3.5) then indicates that t_A is independent of initiation, as one would expect, and the total rate of attrition is thus

$$\phi(t) = r_0 e^{-r_0 t}(1 - F_0(t)) + \int_0^t e^{-r_0 t_0} f_0(t_0)\phi_A(t_A)dt_0, \qquad t_0 = t - t_A, \tag{4.3}$$

where from (3.5)

$$\phi_A(t_A) = \int_0^{T_a/t_A} f_X(x) r(xt_A) \exp\left(-\frac{1}{x}\int_0^{xt_A} r(c)dc\right)dx$$

$$+ \frac{1}{T_a} f_X\left(\frac{T_a}{t_A}\right) \exp\left(-\frac{t_A}{T_a}\int_0^{T_a} r(c)dc\right). \tag{4.4}$$

Because renewals affect only the initial life,[5] the form (4.2) may be immediately applied when there are inspections or renewals.

4.1 Moment Generating Function of Crack Time

This is the Laplace transform of (4.4) which in general is difficult to simplify. To investigate the effect of small departures from translatable cracking, we shall now assume that X is log-Normal. Then, for x near 1, we shall try to relate the extended MGF M_A to that of the median crack M_a.

From the Laplace transform of (4.4)

$$M_A(u) = \int_0^\infty \int_0^{T_a/t_A} f_X(x) r(t_a) \exp\left(-\frac{1}{x}\left[ut_a + \int_0^{t_a} r\,dc\right]\right)dx\,dt_A$$

$$+ \int_0^\infty \frac{1}{T_a} f_X\left(\frac{T_a}{t_A}\right) \exp\left(-\frac{t_A}{T_a}\left[uT_a + \int_0^{T_a} r\,dc\right]\right)dt_A$$

In both terms put $x = e^\xi$, $t_A = t_a e^{-\xi}$ with the Jacobian $J = 1$. In the first term, the upper limit of $x = T_a/t_A$ from which we need ξ. On the right, we have

$$x = T_a e^\xi / t_a \qquad \text{leaving} \qquad t_a \equiv T_a$$

with this upper limit for any value of ξ. Thus, the range of ξ is $(-\infty, \infty)$ whilst $0 < t_a < T_a$.

In the second term, $x \equiv T_a/t_A$ for all t_A, and with this we find $t_a = t_A x \equiv T_a$; that is, the range of t_a is just the point T_a. On the other hand, this identity and the fact that $0 < t_A < \infty$ means that x has the same range. Thus, in the runaway term, the two substitutions have changed an integration over t_A to an averaging over ξ. It would be possible to avoid some of this manip-

ulation if the MGF were originally defined as a Stieltjes integral with a concentrated probability of runaway cracking.

With these changes

$$M_A(u) = \int_0^{T_a} \int_{-\infty}^{\infty} r(t_a) e^{-e^{\xi} G(t_a)} dF_{\Xi}(\xi) dt_a$$

$$+ \int_{-\infty}^{\infty} e^{-e^{\xi} G(T_a)} dF_{\Xi}(\xi), \qquad (4.5)$$

where

$$G(t_a) = ut_a - \int_0^{t_a} r(c)\,dc.$$

4.2 Crack Life MGF for Small Variance

Now assume that $\xi \sim N(0, \sigma^2)$ and expand about $\xi = 0$. In this section and the next steps to successively cruder but more practical approximations are marked by equations. If we use

$$G(t_a) = ut_a - H(t_a)$$

and take three terms of $G \exp - \xi$, then

$$M_A(u) \approx \int_0^{T_a} r(t_a) e^{ut_a - H(t_a)} \exp\left(\frac{\sigma^2 G^2/2}{1 - \sigma^2 G}\right)(1 - \sigma^2 G)^{-1/2} dt_a$$

$$+ e^{G(T_a)} \exp\left(\frac{\sigma^2 G/2}{1 - \sigma^2 G}\right)_{T_a} (1 - \sigma^2 G(T_a))^{-1/2}. \qquad (4.6)$$

It is now convenient to subsume these two terms into one Stieltjes integral over a semi-infinite range with the probability density $dF_a(t_a)$. The terms in (4.6) containing variance expand as

$$\exp\left(\frac{\sigma^2 G^2/2}{1 + \sigma^2 G}\right)\Big/(1 - \sigma^2 G) = 1 + \tfrac{1}{2}\sigma^2(H^2 - H) + \tfrac{1}{4}\sigma^4(\tfrac{3}{2}H^2 - H^3 + \tfrac{1}{2}H^4) + \ldots$$

$$+ ut_a\{1 + \tfrac{1}{2}\sigma^2(H^2 - 3H + 1) + \tfrac{1}{4}\sigma^4(-3H + 4\tfrac{1}{2}H^2 - 3H^3 + \tfrac{1}{2}H^4) + \ldots\}$$

$$+ \tfrac{1}{2}u^2 t_a^2\{1 + \tfrac{1}{2}\sigma^2(H^2 - 3H + 2) + \tfrac{1}{4}\sigma^4(3 + \tfrac{3}{2}H^2 + 3H^3 + \tfrac{1}{2}H^4) + \ldots\}$$

$$+ 0(u^3 t_a^3/6). \qquad (4.7)$$

Here the sum of expectations of the first term on each line obviously form $M_a(u)$, generating moments of the median crack life. If $u = 0$, the kernel corresponding to the first line would (if complete) be the probability density (3.5) of our present model. For M_A, the expectation, however, is taken over median crack lives; the first term is one, so that the variance terms on the first line have zero expectations. In (4.7) therefore, multiples of the variance terms of the first line may be subtracted from the others within an accuracy $0(\sigma^6)$. This simplifies the approximation to

$$M_A(u) \approx M_a(u) - u\int_0^{\infty} t_a\{\tfrac{1}{2}\sigma^2(1 - 2H) - \tfrac{1}{2}\sigma^4(\tfrac{3}{2}(H^2 - H) - H^3)\}dF_a(t_a)$$

$$+ \tfrac{1}{2}u^2 \int_0^{\infty} t_a^2\{\sigma^2(1 - H) - \sigma^4(\tfrac{3}{4} - H^3)\}dF_a(t_a) \qquad (4.8)$$

where the MGF of the median life

$$M_a(u) = \int_0^{\infty} e^{-ut_a} dF_a(t_a).$$

Obviously, the u, u^2 terms in (4.8) represent increases in moments caused by the Normal scatter of crack rates. Similarly, the first line of (4.7) can be interpreted as the corresponding change in crack life density. Although all the integrals converge the errors introduced in (4.6) by the quadratic approximation for $\exp -\xi$ will be large unless $\sigma < 1$ which is true for fatigue.

4.3 Heuristic Approximations of Crack Life Moments

In this section, we derive a rule of thumb to estimate the effect of crack rate variability. To proceed further from (4.7) or (4.8) it is necessary to avoid higher moments of F_a. These are therefore replaced by the corresponding cumulants.

We could also bound the risk as

$$H(t_a) \leqslant g t_a/\mu_a = h t_a, \text{ say,} \tag{4.9}$$

and find strict bounds for the moments. This amounts to approximating the given crack-life risk problem by a constant risk problem. Since these bounds are likely to be very conservative, we shall abandon rigour henceforth and regard $g = 0(1)$ as an averaging correction, retaining all signs. This will provide heuristic estimates of changes and indicate their possible importance.

Treating (4.9) as a simple substitution

$$\begin{aligned} M_A(u) \sim M_a(u) - u\{\tfrac{1}{2}\sigma^2\mu_a - \sigma^2 h(1-\tfrac{3}{4}\sigma^2)\mu_{2a} - \tfrac{3}{4}h^2\sigma^4\mu_{3a} + \tfrac{1}{2}h^3\sigma^4\mu_{4a}\} \\ + \tfrac{1}{2}u^2\{\sigma^2(1-\tfrac{3}{4}\sigma^2)\mu_{2a} - h\sigma^2\mu_{3a} - h^3\sigma^4\mu_{5a}\}. \end{aligned}$$

Now substitute cumulant formulae for the moments μ_{ra} with a view to then neglecting those above κ_2 (we retain the notation μ_a, ${\sigma_a}^2$ for the first two cumulants). This is done because cumulants, unlike moments, can decrease with order. Thus, neglecting higher cumulants and terms in σ^6 and higher, we find

$$\begin{aligned} M_A(u) \sim\ & M_a(u) \\ & - u\{\mu_a[\tfrac{1}{2}\sigma^2 - g\sigma^2(1-\tfrac{3}{4}\sigma^2(1-g)) + \tfrac{1}{2}g^3\sigma^4] + ({\sigma_a}^2/\mu_a)[-g\sigma^2(1-\tfrac{3}{4}\sigma^2) - 2\cdot 25g^2\sigma^4 + 3g^3\sigma^4] \\ & \qquad + \tfrac{3}{2}g^3\sigma^4\sigma_a(\sigma_a/\mu_a)^3\} \\ & + \tfrac{1}{2}u^2\{({\mu_a}^2 + {\sigma_a}^2)[\sigma^2(1-\tfrac{3}{4}\sigma^2) - g\sigma^2 - g^3\sigma^4] - {\sigma_a}^2[2g\sigma^2 + 9g^3\sigma^4 + 15g^3\sigma^4(\sigma_a/\mu_a)^2]\} \end{aligned} \tag{4.10}$$

where $g \approx 1$. If we now make all the gs one, this eventually leads to

$$\begin{aligned} \mu_A/\mu_a &\sim 1 + 0\cdot 6\sigma^2(1+\sigma^2) - (\sigma_a/\mu_a)^2[1 - 1\cdot 5\sigma^2(1 + (\sigma_a/\mu_a)^2)] \\ (\sigma_A/\sigma_a)^2 &\sim 1 - \sigma^2(5 - \sigma^2[2\cdot 25 + (\sigma_a/\mu_a)^2\{14 - 3(\sigma_a(\mu_a)^2\}]) \end{aligned} \tag{4.11}$$

where σ^2 is the logarithmic variance of crack growth rates, while μ_a and σ_a refer to reliability type crack lives based on the median crack growth curve.

4.3.1 Example

Equation (4.11) has been derived from reliability based lives for structures which crack immediately. For the general two-stage model of fatigue it indicates the effect of random rates on cracking times.

The model of crack rates is the coarse randomness in which the possible crack trajectories are randomly scaled versions of the same median curve. As a typical example, put

$$\sigma = 0\cdot 2303, \qquad \sigma_a/\mu_a = (e^{\sigma^2} - 1) = 0\cdot 2589$$

which corresponds to a log-Normal life, with logarithmic standard deviation[6] $0\cdot 1$ (to base 10). To base 10 the crack rate standard deviation[7] is also $0\cdot 1$; the quantities in (4.11) refer to natural logarithms.

After substitution, we find $\mu_A/\mu_a = 1 - 0{\cdot}03344$, $(\sigma_A/\sigma_a)^2 = 1 - 0{\cdot}25617$ and $\sigma_A/\sigma_a = 1 - 0{\cdot}13754$.

The general effect indicated seem to be a small reduction in mean crack time and a possibly more important reduction of its variance.

5. CONCLUSIONS

The methods of Reference 1 have been applied to one-crack models in which all the crack trajectories are smooth, but form a family in which the growth rate for a given crack length is a random factor of the median rate. The application is straightforward and it is simple to allow for runaway cracks. There is no simple relation to the results for median crack lives, although asymptotic effects on the moments are suggested.

Application to a typical example seems to indicate that the mean life with random cracking is slightly less than that from the median crack, but there is a more important reduction of variance. These results are being sharpened by further study of the asymptotic moment generating function.

The model described above refers to structures in which cracking begins immediately. In Section 4 the generalised density corresponding to this has been used to extend the previous[1] two-stage model. This allows randomness in both the initiation and crack growth stages.

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